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Deep Learning with Softmax and SVM using Worst Omega Optimization for Multi-Class Financial Prediction

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ABSTRACT

This study proposes a hybrid framework for multi-class financial prediction and portfolio optimization by integrating deep learning-based classification models with worst-case Omega optimization. The framework employs a neural network architecture combined with Softmax and multi-class Support Vector Machine (SVM) classifiers to categorize assets into low-, medium-, and high-return classes. These classifications are subsequently utilized to construct portfolios using a worst-case Omega optimization model that explicitly accounts for downside risk and uncertainty. The empirical analysis is conducted on two benchmark datasets, BSE 30 and DOW 30, using a rolling window approach. The results demonstrate that the SVM-based classification model outperforms the Softmax model in terms of class separability and stability across varying market conditions. Portfolios constructed using SVM-selected assets consistently achieve higher returns, lower volatility, and improved risk-adjusted performance, as evidenced by superior Sharpe, Sortino, STARR, and Omega ratios. Furthermore, the worst-case Omega optimization framework provides enhanced protection against extreme losses by effectively controlling tail risk, as reflected in lower Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). Comparative analysis with equally weighted portfolios confirms the ability of the proposed framework to generate persistent excess returns across different risk-aversion levels. Overall, the study highlights the importance of combining accurate classification techniques with robust optimization methods for effective portfolio management. The proposed approach offers a flexible and reliable solution for decision-making in dynamic and uncertain financial markets.

1. INTRODUCTION

Financial markets are inherently complex, dynamic ,and characterized by nonlinearity and uncertainty.The problem of predicting asset returns and constructing optimal portfolios remains a fundamental challenge in quantitative finance. The classical mean-variance framework [1], [2] has been widely used; however, its reliance on normality assumptions and variance as a risk measure limits its effectiveness in real-world markets. Recent advancements in deep learning have significantly improved financial prediction by capturing nonlinear dependencies and temporal structures in financial data [3], [4], [5]. Hybrid machine learning frameworks have further enhanced prediction accuracy by combining different models and feature extraction techniques [6], [7]. In financial prediction, classification-based approaches have gained importance as they categorize assets into return-based groups, enabling better decision-making compared to regression models [8]. Softmax classifiers are widely used due to their probabilistic nature, whereas Support Vector Machines (SVM) offer superior generalization and robustness [9].

In practical portfolio management, investors face two major challenges: accurately identifying high-return assets under highly volatile market conditions and constructing portfolios that remain robust during extreme downside events. Traditional regression-based prediction models often fail to provide clear asset grouping for decision-making, while classical optimization models inadequately capture tail risk and uncertainty. This creates a need for an integrated framework that combines predictive intelligence with robust portfolio optimization. The proposed methodology addresses this challenge by using multi-class classification for asset selection and worst-case Omega optimization for downside risk control, thereby improving both return generation and portfolio resilience in dynamic financial markets.

Portfolio optimization has also evolved significantly with the integration of artificial intelligence. Machine learning enhanced portfolio strategies have demonstrated improved risk-adjusted performance and better asset

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allocation decisions [10], [11]. However, traditional risk measures fail to capture tail risk effectively. The Omega ratio provides a comprehensive performance measure by considering the entire return distribution [12]. Unlike variance-based metrics, it captures higher-order moments and downside risk [13]. Furthermore, worst-case Omega optimization enhances robustness by accounting for uncertainty and extreme market conditions [14]. Recent studies have explored Fuzzy SVM [15], [16], Fuzzy Least Square SVM(LSSVM) [17], Multi Class SVM (MSVM) [18], deep reinforcement learning, and hybrid AI models for portfolio optimization, demonstrating superior performance in dynamic markets [19]; [20], [21], [22]. These models improve adaptability, risk management, and return generation.

1.1. Literature Review

The integration of artificial intelligence into portfolio optimization has gained significant attention in recent years. Early studies focused on combining machine learning with traditional optimization techniques [23]. However, recent research emphasizes deep learning-based prediction and optimization. Deep learning models such as LSTM, CNN, and hybrid architectures have demonstrated strong predictive capabilities in financial markets [4], [6], [7]. Clustering-based deep learning approaches have further improved return prediction and portfolio performance [6]. Reinforcement learning has emerged as a powerful approach for portfolio optimization. Recent studies propose dynamic portfolio strategies using deep reinforcement learning frameworks, achieving higher cumulative returns and better risk control [19], [20], [22]. Charkhestani and Esfahanipour [22] introduced a behaviorally informed deep reinforcement learning framework by incorporating investor psychology factors such as loss aversion and overconfidence into dynamic portfolio optimization. Hybrid frameworks combining LSTM and reinforcement learning have also shown improved adaptability and robustness [24]. Advanced optimization techniques, including distributionally robust optimization and worst-case optimization, have been introduced to address uncertainty in financial markets [14], [25]. These methods improve downside risk management and portfolio resilience. Ren et al. [26] proposed a hierarchical deep reinforcement learning framework with segmented allocation and momentum-adjusted utility for multi-agent portfolio management, improving scalability and decision-making efficiency. Similarly, Ranabhat et al. [27] applied variational neural annealing to large-scale portfolio optimization to enhance optimization efficiency and computational scalability. Despite these advancements, most studies focus on either prediction or optimization independently. Unlike these recent reinforcement learning and variational optimization approaches, the proposed framework integrates deep learning-based multi-class classification using comparative Softmax and multi-class SVM models and directly links classification outputs to worst-case Omega optimization. This provides both stronger predictive discrimination and robust downside risk control within a unified portfolio decision-making framework.

1.2. Research Gap

Although significant progress has been made, several gaps remain. First, most studies rely on regression-based prediction rather than classification-based frameworks. Second, multi-class classification approaches for financial prediction remain underexplored. Third, Softmax classifiers are widely used, but their limitations in handling class imbalance and reduced discriminative power are not adequately addressed [28]. Comparative analysis with SVM in financial contexts is limited. Fourth, traditional optimization models fail to capture tail risk effectively. Although Omega-based optimization has been studied [29], its integration with classification-based asset selection is still lacking. Finally, there is a lack of unified frameworks combining deep learning, classification, and worst-case optimization.

1.3. Motivation

This study is motivated by the need to develop a unified framework that integrates prediction accuracy with robust portfolio optimization. Financial markets are highly volatile, requiring models that balance return generation and risk control. By combining deep learning with multi-class classification, this study aims to improve asset selection.The inclusion of SVM enhances classification robustness. Furthermore, worst-case Omega optimization enables effective downside risk management.

1.4. Contributions and Novelty

The main contributions are as follows. First, a hybrid framework integrating deep learning with multi-class classification is proposed. Second, a comparative analysis of Softmax and SVM is conducted. Third, worst-case Omega optimization is incorporated to address tail risk. Fourth, the framework is validated using real-world datasets. Finally, the study demonstrates improved portfolio performance and robustness.

Unlike existing studies that separately focus on either prediction models or optimization frameworks, this study develops a fully integrated pipeline where deep learning-based classification directly drives portfolio construction. The novelty lies in comparing Softmax and multi-class SVM within the same predictive architecture and linking their outputs to worst-case Omega optimization. This allows simultaneous improvement in classification quality, downside risk management, and portfolio robustness, which remains limited in existing literature.

1.5. Organization of the Paper

The remainder of the paper is organized as follows. Section 2 presents the mathematical formulation. Section 3 describes the methodology. Section 4 presents the results. Section 5 concludes the study.

2. PRELIMINARIES

This section presents the mathematical foundations of the proposed framework, which integrates deep neural network-based classification with portfolio optimization. The formulation consists of two main components: neural network representation and classification via Softmax and multi-class SVM.

2.1. Neural Network Representation

Let $X \in \mathbb{R}^{N \times D}$ denote the input feature matrix, where N represents the total number of observations and D denotes the number of input features associated with each observation. The neural network architecture considered in this study consists of a single hidden layer with H neurons, where H controls the model's capacity to learn complex patterns, and an output layer corresponding to K classes, where K represents the number of asset categories.

The hidden layer is parameterized by a weight matrix $W_1 \in \mathbb{R}^{D \times H}$, which maps input features into the hidden representation space, and a bias vector $b_1 \in \mathbb{R}^{1 \times H}$, which allows shifting of the activation function. The pre-activation values are computed as

$$z^{(1)} = XW_1 + b_1, \quad h = \max\left(0, z^{(1)}\right) \qquad (1)$$

where $z^{(1)}$ represents the linear transformation of inputs, and $h \in \mathbb{R}^{N \times H}$ denotes the hidden layer output after applying the ReLU activation function. The ReLU function introduces non-linearity and helps in learning complex relationships while avoiding vanishing gradient issues.

The output layer is defined by a weight matrix $W_2 \in \mathbb{R}^{H \times K}$, which projects the hidden features into class scores, and a bias vector $b_2 \in \mathbb{R}^{1 \times K}$. The resulting class score matrix is computed as

$$a = hW_2 + b_2 \qquad (2)$$

where $a \in \mathbb{R}^{N \times K}$, and each element $a_{i,k}$ represents the score assigned by the model to class k for observation i. These scores are later used differently by Softmax and SVM classifiers.

2.2. Softmax-Based Classification Model

The Softmax-based classification model transforms the raw class scores into probabilistic outputs, enabling interpretation in terms of class membership likelihood. Specifically, for each observation i, the exponential function is applied to each class score $a_{i,k}$ and normalized across all K classes to produce probabilities. The model parameters W_1, W_2, b_1, and b_2 are jointly optimized by minimizing the regularized cross-entropy loss.

$$\min_{W_1,W_2,b_1,b_2} -\sum_{i=1}^{N} \log\left(\frac{e^{a_{i,y_i}}}{\sum_{j=1}^{K} e^{a_{i,j}}}\right) + \lambda(\| W_1 \|_F^2 + \| W_2 \|_F^2) \qquad (3)$$

In this formulation, y_i denotes the true class label of observation i, and the loss function penalizes the model when the predicted probability of the correct class is low. The regularization parameter λ controls the trade-off between fitting the data and preventing overfitting by penalizing large weights through the Frobenius norms $\| W_1 \|_F^2$ and $\| W_2 \|_F^2$.

Prediction is given by:

$$\hat{y}_i = \underset{k}{\operatorname{argmax}} \frac{e^{a_{i,k}}}{\sum_j e^{a_{i,j}}} \qquad (4)$$

This rule assigns each observation to the class with the highest predicted probability. The Softmax model is particularly suitable when probabilistic interpretation and smooth decision boundaries are required.

2.3. Multi-Class Support Vector Machine Model

The multi-class Support Vector Machine (SVM) model uses the same neural network-generated scores $a_{i,k}$ but adopts a different objective function based on margin maximization. Instead of probabilities, the SVM focuses on ensuring that the score of the correct class a_{i,y_i} is sufficiently larger than the scores of all other classes $a_{i,j}$ for $j \neq y_i$.

$$\min_{W_1,W_2,b_1,b_2} \sum_{i=1}^{N} \sum_{j \neq y_i} \max\left(0,\ 1 - a_{i,y_i} + a_{i,j}\right) + \lambda(\| W_1 \|_F^2 + \| W_2 \|_F^2) \qquad (5)$$

Here, the hinge loss term $\max(0, 1 - a_{i,y_i} + a_{i,j})$ penalizes violations of the margin condition. If the difference between the correct class score and an incorrect class score is less than 1, a loss is incurred; otherwise, no penalty is applied. This encourages the model to learn well-separated class boundaries. The regularization term controlled by λ again prevents overfitting.

Prediction rule:

$$\hat{y}_i = \underset{k}{\operatorname{argmax}}\, a_{i,k} \qquad (6)$$

In this case, the predicted class is simply the one with the highest score. Unlike Softmax, SVM does not produce probabilities but typically yields sharper and more robust decision boundaries, which is advantageous in noisy financial datasets.

2.4. Worst-Case Omega Portfolio Optimization Model

Let $x \in \mathbb{R}^n$ denote the portfolio weight vector, where n is the number of assets and each element x_i represents the proportion of total capital allocated to asset i. The optimization model integrates multiple components, including expected returns, prediction-based signals, and multi-horizon returns, to construct a robust portfolio.

$$\begin{aligned} \max_{x,\psi,\eta} \quad & k_1\psi + k_2(x^\top\mu) + k_3(x^\top\bar{E}) + k_4(x^\top R^L) + k_5(x^\top R^S) \\ \text{s.t.} \quad & \sum x_i = 1,\ 0 \le x_i \le 1 \\ & \delta(x^\top e_t) - \frac{1-\delta}{\tau}\sum_j \eta_{j,t} \ge \psi \\ & \eta_{j,t} \ge -x^\top e_t,\ \eta_{j,t} \ge 0 \end{aligned} \tag{7}$$

In this formulation, $\mu \in \mathbb{R}^n$ represents the expected return vector, $\bar{E} \in \mathbb{R}^n$ captures prediction errors or model-based signals, R^L and R^S denote long-term and short-term return vectors, respectively, and $k_1, \ldots, k_5$ are weighting parameters that balance the importance of each component in the objective function. The variable ψ represents the worst-case performance measure (Omega-based threshold), while $\eta_{j,t}$ are auxiliary slack variables introduced to linearize downside risk constraints across scenarios indexed by j and time t.

The constraints ensure full investment ($\sum x_i = 1$) and no short-selling ($0 \le x_i \le 1$). The parameter $\delta \in [0,1]$ controls the trade-off between expected return and downside risk, while τ represents the number of scenarios considered. This formulation enhances robustness by explicitly accounting for adverse market conditions, making it particularly suitable for financial portfolio optimization under uncertainty.

3. Methodology

This section presents the proposed framework for asset classification and portfolio optimization. The methodology integrates deep learning-based predictive modeling with a worst-case Omega portfolio optimization, providing a systematic approach to select high-performing assets and construct robust portfolios.

3.1. Return Representation

We begin by transforming asset price series into returns, which serve as the fundamental input for both predictive modeling and risk assessment. Returns are computed as the one-period relative change in price:

$$r_{t,i} = \frac{P_{t,i} - P_{t-1,i}}{P_{t-1,i}} \tag{8}$$

where $P_{t,i}$ denotes the price of asset i at time t. The resulting return matrix $R \in \mathbb{R}^{T\times N}$ captures both temporal and cross-sectional variations across N assets over T time periods.

3.2. Dataset Description and Preprocessing

The study utilizes two major equity datasets: BSE 30 (India) and DOW 30 (USA). Asset identifiers and corresponding company names are summarized in Tables 1 and 2. Descriptive statistics (Tables 3 and 4) indicate notable non-normality, high skewness, and volatility clustering, which motivate the inclusion of features that capture both trend and risk dynamics. Moreover, the data used in this study is of a 7-year (≈ 1800 trading days) timespan consisting of the period from January 2015 to March 2022. Also, the data is publicly accessible on Yahoo! Finance at https://finance.yahoo.com/.

Table 1. BSE 30 Companies (India)

Asset	Company Name
A1	Asian Paints Ltd.
A2	Axis Bank Ltd.
A3	Bajaj Finance Ltd.
A4	Bajaj Finserv Ltd.
A5	Bharti Airtel Ltd.
A6	HCL Technologies Ltd.
A7	HDFC Bank Ltd.
A8	Hindustan Unilever Ltd.
A9	ICICI Bank Ltd.
A10	IndusInd Bank Ltd.
A11	Infosys Ltd.
A12	ITC Ltd.
A13	JSW Steel Ltd.
A14	Kotak Mahindra Bank Ltd.
A15	Larsen & Toubro Ltd.
A16	Mahindra & Mahindra Ltd.
A17	Maruti Suzuki India Ltd.
A18	Nestlé India Ltd.
A19	NTPC Ltd.
A20	Power Grid Corporation of India Ltd.
A21	Reliance Industries Ltd.
A22	State Bank of India
A23	Sun Pharmaceutical Industries Ltd.
A24	Tata Consultancy Services Ltd.
A25	Tata Motors Ltd.
A26	Tata Steel Ltd.
A27	Tech Mahindra Ltd.
A28	Titan Company Ltd.
A29	UltraTech Cement Ltd.
A30	Wipro Ltd.

3.3. Feature Engineering

To enrich the raw return series for predictive modeling, we construct a comprehensive set of features for each asset. These features aim to capture temporal dependencies, short-term momentum, volatility clustering, and market-wide risk. Table 5 summarizes the engineered features.

For all assets, these features are concatenated to form a panel dataset:

$$\mathcal{D} = \{ (X_{t,i}, y_{t,i}) \mid t = 1, \ldots, T,\ i = 1, \ldots, N \}, \tag{9}$$

where $X_{t,i} \in \mathbb{R}^d$ is the feature vector and $y_{t,i}$ is the prediction target. Missing values from lagging and rolling operations are removed to ensure consistency.

This feature set provides the neural network with rich information to model:

Temporal dynamics via lagged returns and rolling statistics,

Market-wide risk via cross-sectional volatility,

Volatility persistence through rolling volatility clusters,

Short-term trends using rolling averages.

3.4. *Target Construction and Labeling*

The predictive target is the one-step-ahead return:

$$y_{t,i} = r_{t+1,i} \tag{10}$$

To enable classification-based portfolio decisions, the continuous return is discretized into three quantile-based classes:

$$y_{t,i} = \begin{cases} 0, & y_{t,i} \le q_1, \\ 1, & q_1 < y_{t,i} \le q_2, \\ 2, & y_{t,i} > q_2, \end{cases} \tag{11}$$

where q_1 and q_2 are the 33rd and 66th percentiles, respectively. Class 1 and Class 2 represent middle and high-return assets, which are prioritized in the portfolio construction stage.

Table 2. DOW 30 Companies (USA)

Asset	Company Name
B1	3M Company
B2	American Express Company
B3	Amgen Inc.
B4	Apple Inc.
B5	Boeing Company
B6	Caterpillar Inc.
B7	Chevron Corporation
B8	Cisco Systems, Inc.
B9	The Coca-Cola Company
B10	Goldman Sachs Group, Inc.
B11	The Home Depot, Inc.
B12	Honeywell International Inc.
B13	Intel Corporation
B14	International Business Machines Corporation (IBM)
B15	Johnson & Johnson
B16	JPMorgan Chase & Co.
B17	McDonald's Corporation
B18	Merck & Co., Inc.
B19	Microsoft Corporation
B20	Nike, Inc.
B21	Procter & Gamble Company
B22	Salesforce, Inc.
B23	The Travelers Companies, Inc.
B24	UnitedHealth Group Incorporated
B25	Verizon Communications Inc.
B26	Visa Inc.
B27	Walgreens Boots Alliance, Inc.
B28	Walmart Inc.
B29	The Walt Disney Company

Table 3. Descriptive Statistics of BSE 30 Assets

Asset	Mean	Std	Max	Min	Range	Kurtosis	Skewness
A1	1545.91	733.27	3574.65	699.55	2875.10	0.24	1.16
A2	585.08	116.80	845.05	303.15	541.90	-1.02	0.26
A3	2696.28	2041.21	7927.70	342.58	7585.13	-0.13	0.90
A4	628.32	422.48	1905.41	122.85	1782.56	1.01	1.18
A5	429.12	112.27	764.95	285.90	479.05	0.30	1.07
A6	591.86	244.37	1357.65	354.38	1003.28	1.00	1.50
A7	973.55	341.54	1688.95	471.33	1217.62	-1.09	0.19
A8	1559.30	599.75	2812.05	756.20	2055.85	-1.42	0.14
A9	379.99	157.59	841.05	166.68	674.37	0.36	1.17
A10	1196.78	391.79	2021.55	301.20	1720.35	-0.76	0.08
A11	790.34	385.68	1939.35	436.75	1502.60	0.98	1.50
A12	240.43	36.02	342.30	147.20	195.10	-0.61	0.08
A13	282.59	175.02	763.20	81.11	682.09	0.79	1.26
A14	1205.02	429.87	2210.55	599.35	1611.20	-1.14	0.30
A15	1220.63	271.60	2069.35	702.47	1366.88	0.10	0.55
A16	682.64	119.63	982.40	269.25	713.15	0.61	-0.06
A17	6436.02	1653.44	9820.35	3242.60	6577.75	-0.88	-0.21
A18	1100.11	479.18	2043.94	501.23	1542.71	-1.35	0.46
A19	121.48	16.58	155.04	76.20	78.84	-0.53	-0.44
A20	106.20	19.30	164.06	68.63	95.44	0.24	0.39
A21	1177.09	655.37	2731.50	405.17	2326.33	-0.90	0.59
A22	290.34	79.10	540.45	150.85	389.60	1.04	1.04
A23	634.36	170.05	1168.50	324.00	844.50	-0.82	0.39
A24	1936.35	792.03	4019.10	1052.53	2966.58	-0.20	0.94
A25	327.54	141.80	605.10	65.30	539.80	-1.21	-0.11
A26	55.06	29.18	151.91	20.13	131.78	1.57	1.51
A27	714.87	295.97	1806.25	376.45	1429.80	2.43	1.65
A28	938.38	571.24	2657.35	303.05	2354.30	0.72	1.06
A29	4299.97	1329.68	8212.55	2602.65	5609.90	0.92	1.33
A30	282.69	134.79	721.55	161.95	559.60	2.13	1.83

Table 4. Descriptive Statistics of DOW 30 Assets

Asset	Mean	Std	Max	Min	Range	Kurtosis	Skewness
B1	151.49	20.38	216.25	98.55	117.69	-0.21	0.52
B2	100.57	29.80	187.08	51.11	135.97	0.18	0.86
B3	190.14	32.04	260.95	132.24	128.71	-0.97	0.47
B4	61.11	39.99	180.33	22.58	157.75	0.08	1.20
B5	231.82	90.14	440.62	95.01	345.61	-1.30	0.43
B6	128.20	44.60	244.79	57.91	186.88	-0.29	0.64
B7	105.67	14.03	133.60	54.22	79.38	-0.20	-0.58
B8	40.08	9.99	63.42	22.51	40.91	-1.25	0.14
B9	47.03	5.03	60.13	37.56	22.57	-0.74	0.57
B10	228.79	63.11	423.85	134.97	288.88	1.57	1.45
B11	195.63	70.73	416.18	100.95	315.23	0.08	0.87
B12	146.44	39.67	234.18	87.32	146.86	-0.64	0.56
B13	44.91	10.32	68.47	25.87	42.60	-1.20	0.07
B14	137.07	14.93	173.95	90.60	83.35	-0.39	0.01
B15	131.67	20.59	179.47	90.73	88.74	-0.71	0.03
B16	101.01	30.58	171.78	53.07	118.71	-0.59	0.43
B17	166.77	45.91	268.24	88.78	179.46	-1.12	0.04
B18	65.85	11.01	90.54	46.20	44.34	-1.39	0.10
B19	124.46	78.82	343.11	40.29	302.82	-0.04	1.00
B20	83.22	33.99	177.51	45.58	131.92	0.23	1.14
B21	101.71	23.35	161.97	68.06	93.91	-1.05	0.62
B22	138.47	64.91	309.96	54.05	255.91	-0.60	0.70
B23	126.51	16.97	162.58	81.69	80.89	-0.81	0.22
B24	233.80	94.46	499.50	98.92	400.58	-0.44	0.55
B25	52.86	4.75	62.07	42.84	19.23	-1.15	-0.06
B26	137.25	55.64	250.93	61.59	189.34	-1.34	0.27
B27	66.69	16.25	96.68	33.52	63.16	-1.29	-0.25
B28	32.98	9.19	50.93	18.81	32.12	-1.16	0.41
B29	121.80	26.75	201.91	85.76	116.15	0.39	1.22

Table 5. Engineered Features for Each Asset

Feature	Description
Mean	Contemporaneous return $r_{t,i}$
Std	Cross-sectional standard deviation of returns at time t
lag_1 – lag_5	Lagged returns $r_{t-1,i}, \dots, r_{t-5,i}$ capturing short-term dependencies
roll_mean_5	5-day rolling mean of returns (short-term trend)
roll_std_5	5-day rolling standard deviation (short-term volatility)
vol_cluster	10-day rolling average of cross-sectional volatility, capturing volatility persistence

3.5. Deep Neural Network Model

The neural network transforms feature vectors $X_{t,i}$ into latent representations through hidden layers:

$$z^{(1)} = XW_1 + b_1, \tag{12}$$

$$h = \sigma(z^{(1)}), \tag{13}$$

$$a = hW_2 + b_2, \tag{14}$$

where σ is a ReLU activation. These latent representations form the basis for subsequent classification with Softmax or SVM layers.

3.6. Softmax and SVM-Based Classification

The Softmax layer predicts class probabilities, while the SVM computes decision boundaries for classification. Loss functions for both models are minimized during training:

$$\begin{aligned} \mathcal{L}_{\text{Softmax}} &= -\sum_i y_i \log \hat{y}_i, \\ \mathcal{L}_{\text{SVM}} &= \sum_{i=1}^{N} \sum_{j \neq y_i} \max\left(0,\ 1 - a_{i,y_i} + a_{i,j}\right) + \lambda(\| W_1 \|_F^2 + \| W_2 \|_F^2) \end{aligned} \tag{15}$$

Predictions $\hat{y}_{t,i}$ are then used for asset selection. The complete classification procedure is summarized in Algorithm 1.

3.6.1. Handling Class Imbalance and Comparative Robustness of Softmax and SVM

The Cross-entropy loss in Softmax measures the difference between the true class label and the predicted class probability distribution.

For a multi-class problem with C classes, the Softmax probability is:

$$P(y_i = j \mid x_i) = \frac{e^{z_{ij}}}{\sum_{k=1}^{C} e^{z_{ik}}}$$

where z_{ij} is the output score (logit) for class j.

The cross-entropy loss for one sample is:

$$L_i = -\sum_{j=1}^{C} y_{ij} \log(P(y_i = j \mid x_i))$$

where y_{ij} is the one-hot encoded true label.

Since only the true class has value 1, it becomes:

$$L_i = -\log(P(\text{true class}))$$

Thus, if the predicted probability of the correct class is high, loss is low; otherwise, loss is large.

For class imbalance, weighted cross-entropy is used:

$$L_i = -w_{y_i} \log(P(\text{true class}))$$

where w_{y_i} assigns larger penalties to minority classes, improving balanced classification.

In SVM, the objective is not probability estimation like Softmax, but margin maximization between the correct class and incorrect classes.

For multi-class SVM, the hinge loss is:

$$L_i = \sum_{j \neq y_i} \max\left(0,\ 1 + z_{ij} - z_{i,y_i}\right)$$

where:

z_{i,y_i} = score of the true class

z_{ij} = score of the wrong class

margin = difference between correct and incorrect class scores

If the correct class score is sufficiently larger than the others (by at least 1), the loss becomes 0.

Otherwise, a penalty is applied.

Thus, SVM learns sharper decision boundaries by forcing better class separation, making it more robust than Softmax under noisy and imbalanced financial datasets.

3.7. Rolling Window Framework

To capture evolving market dynamics and prevent overfitting to a specific period, a rolling window framework is employed. The dataset is divided into sequential windows, with the model retrained and tested on each window. This ensures robust out-of-sample evaluation while reflecting changing market conditions. In this study, we use five rolling windows. Each window consists of:

Training period: 500 observations

Testing period: 250 observations

Step size: 250 observations (non-overlapping test sets)

Table 6 summarizes the start and end points of each rolling window, clearly indicating the training and testing periods.

Table 6. Rolling Window Setup for Model Training and Testing

Window	Training Period	Testing Period	Step Size
1	1 – 500	501 – 750	250
2	251 – 750	751 – 1000	250
3	501 – 1000	1001 – 1250	250
4	751 – 1250	1251 – 1500	250
5	1001 – 1500	1501 – 1750	250

Each window allows the model to learn from historical data and predict forward returns in the subsequent test set. After each window, the model weights are reset and retrained on the next training set, ensuring that predictions always reflect the most recent market information. This approach enhances the reliability of asset selection and portfolio construction by incorporating temporal adaptivity.

3.8. Asset Selection Strategy

Assets are selected based on predicted class labels:

$$\mathcal{S} = \left\{ i \mid \hat{y}_{t,i} \in \{1,2\} \right\}. \tag{16}$$

This ensures that only assets with predicted positive returns are considered for portfolio allocation.

3.9. Worst-Case Omega Portfolio Optimization

For the selected assets, portfolio weights x^* are determined by maximizing the worst-case Omega ratio:

$$x^* = \underset{x}{\operatorname{argmax}}\, \Omega_{\text{wc}}(x) \quad \text{s.t.} \sum_i x_i = 1,\ x_i \geq 0 \tag{17}$$

Algorithm 2 details the optimization procedure.

3.10. Portfolio Return and Evaluation

The out-of-sample portfolio return is computed as:

$$y_t = R_t^{\text{out}} x^*, \tag{18}$$

where R_t^{out} represents returns of the selected assets in the testing period. This allows for rigorous evaluation of portfolio performance beyond the training sample.

3.11. Integrated Decision Pipeline

The complete framework can be summarized as:

$$X \to h \to \hat{y} \to \mathcal{S} \to x^* \to y_t, \tag{19}$$

highlighting the seamless integration of feature engineering, predictive modeling, asset selection, optimization, and portfolio return computation.

3.12. Algorithmic Framework

The overall methodology is operationalized through two algorithms:

Algorithm 1: Deep learning-based classification

Algorithm 2: Worst-case Omega optimization

This integrated approach ensures that both predictive and optimization stages are aligned to maximize portfolio performance while accounting for market risk.

Algorithm 1: Deep Learning–Based Softmax and SVM Asset Classification

Require: Time series of asset prices $P = \{P_{t,i}\}$; training window length L_train; testing window length L_{test}; step size s; number of classes $K = 3$ learning rate η; epochs E.

Ensure: Predicted class labels Ŷ and class-wise grouped assets.

```
Compute returns $r_{t,i} = \frac{P_{t,i} - P_{t-1,i}}{P_{t-1,i}}$
Construct feature set $X_{t,i}$ using returns, volatility, lags, rolling mean, rolling std, and volatility clustering
Define target $y_{t,i} = r_{t+1,i}$
Convert $y_{t,i}$ into 3 classes using quantiles q1, q2
For each rolling window w do
    Split data into training and testing sets
    Standardize features using training statistics
    Initialize $W_1, W_2 \sim \mathcal{N}(0,0.01)$, $b_1, b_2 = 0$
    Compute class weight $w_i = \frac{1}{\text{frequency of class } i}$
    For epoch = 1 to E do
        $z^{(1)} = XW_1 + b_1$
        $h = \max(0, z^{(1)})$
        $a = hW_2 + b_2$
        If Softmax model then
            $p_k = \frac{e^{a_k}}{\sum_j e^{a_j}}$
            $\frac{\partial L}{\partial a} = w_i(p - y_{\text{one-hot}})$
        Else
            Compute margins $\max(0, 1 - a_{y_i} + a_j)$
```

```
Construct binary mask and compute row sum
Set correct class gradient as negative row sum
Compute $\frac{\partial L}{\partial a}$
End if
Update parameters: $W \leftarrow W - \eta(\nabla W + \lambda W)$   $b \leftarrow b - \eta \nabla b$
End for
Compute test scores
Predict: Softmax: $\hat{y} = \mathrm{argmax}(p)$   SVM: $\hat{y} = \mathrm{argmax}(a)$
Select the latest prediction for each asset
Group assets into classes 0, 1, 2
Store class-wise asset prices
End for
Return $\hat{Y}$
```

Algorithm 2: Worst-Case Omega Portfolio Optimization using Shortlisted Assets

Require: Selected asset set S; rolling windows w; parameters τ, δ; weights $k1, \dots, k5$

Ensure: Optimal weights x^* and portfolio returns

```
For each rolling window $w$ do
Load prices and compute returns $r_{t,i}$
Construct return matrix $R$ and split into training $R$ and testing $R^{out}$
Compute expected return vector $\mu = \mathbb{E}[R]$
Compute prediction errors $E = R - \hat{R}$ and average error $\bar{E}$
Compute long-term and short-term returns $R^L$ and $R^S$
Define scenarios using return vectors $e_t$
Initialize variables $x \in \mathbb{R}^{|S|}$, $\psi \in \mathbb{R}$, $\eta_{j,t}$
Solve optimization problem
Compute out-of-sample returns
Store results
End for
Return portfolios and returns
```

4. EXPERIMENT

This section presents the empirical analysis of the proposed framework, which integrates deep learning-based classification models with worst-case Omega portfolio optimization. The experiment is conducted on two benchmark indices, namely BSE 30 and DOW 30, across multiple rolling windows. The analysis is structured into three main components: asset selection using Softmax and SVM models, followed by portfolio optimization using shortlisted assets.

4.1. Asset Selection using Softmax Model

Tables 7 and 8 present the shortlisted assets obtained from the Softmax classification model across five rolling windows. The assets are categorized into three classes, where Class 0 represents low-return assets, Class 1 represents moderate-return assets, and Class 2 represents high-return assets.

For the BSE 30 dataset, it is observed that the distribution of assets across classes varies significantly across different windows. In the first window, a large proportion of assets are classified into Class 0, indicating a conservative market outlook, while only a few assets are identified as high-return candidates in Class 2. However, in later windows such as W3 and W4, a noticeable shift occurs where a larger number of assets move into Class 2, suggesting improved market conditions and higher return expectations. In W5, almost all assets are concentrated in Class 0, indicating a strong bias of the model toward low-risk or low-return predictions during that period.

Similarly, for the DOW 30 dataset, the Softmax model exhibits distinct classification behavior. In W1, the assets are relatively well distributed across all three classes, indicating balanced market conditions. However, in W2 and W3, the majority of assets are concentrated in a single class (Class 0 and Class 1, respectively), reflecting reduced discrimination capability of the model in those windows. This suggests that the Softmax model may sometimes produce biased classifications under certain market regimes.

The key insight from the Softmax-based asset selection is that while the model is capable of identifying high-return assets, its class distribution is highly sensitive to market dynamics and may exhibit class imbalance in certain periods.

Table 7. Shortlisted Assets for BSE 30 (Softmax Model)

Sheet	Shortlisted Assets
W1_Softmax_C0	A1, A3, A4, A6, A7, A8, A10, A11, A14, A15, A16, A17, A18, A19, A20, A21, A22, A24, A25, A27, A28, A30
W1_Softmax_C1	A2, A13, A29
W1_Softmax_C2	A5, A9, A12, A23, A26
W2_Softmax_C0	A3, A4, A6, A7, A11, A12, A14, A17, A21, A23, A24, A26, A27, A28, A30
W2_Softmax_C1	A1, A2, A5, A8, A9, A10, A13, A15, A16, A18, A19, A20, A22, A25, A29
W3_Softmax_C0	A1, A7, A8, A14, A18, A24, A27, A29, A30
W3_Softmax_C1	A11, A22
W3_Softmax_C2	A2, A3, A4, A5, A6, A9, A10, A12, A13, A15, A16, A17, A19, A20, A21, A23, A25, A26, A28
W4_Softmax_C0	A10, A13, A19, A21, A25, A26
W4_Softmax_C1	A2, A3, A4, A5, A7, A8, A9, A11, A15, A18, A22, A28, A30
W4_Softmax_C2	A1, A6, A12, A14, A16, A17, A20, A23, A24, A27, A29
W5_Softmax_C0	A1, A2, A3, A4, A7, A8, A9, A10, A11, A13, A14, A15, A16, A17, A18, A19, A20, A21, A22, A23, A24, A25, A26, A27, A28, A29, A30
W5_Softmax_C2	A5, A6, A12

Table 8. Shortlisted Assets for DOW 30 (Softmax Model)

Sheet	Shortlisted Assets
W1_Softmax_C0	B2, B3, B5, B7, B11, B14, B15, B19, B21, B23, B26, B28
W1_Softmax_C1	B4, B6, B8, B12, B13, B17, B20, B24, B25
W1_Softmax_C2	B1, B9, B10, B16, B18, B22, B27, B29
W2_Softmax_C0	B1, B2, B3, B5, B6, B7, B8, B9, B10, B11, B12, B13, B14, B15, B16, B17, B18, B19, B20, B21, B22, B23, B24, B25, B26, B27, B28, B29
W2_Softmax_C1	B4
W3_Softmax_C1	B1, B2, B3, B4, B5, B6, B7, B8, B9, B10, B11, B12, B13, B14, B15, B16, B17, B18, B19, B20, B21, B22, B23, B24, B25, B26, B27, B28, B29
W4_Softmax_C1	B1, B2, B4, B5, B6, B7, B8, B9, B11, B12, B14, B15, B17, B18, B19, B21, B22, B23, B24, B25, B28, B29
W4_Softmax_C2	B3, B10, B13, B16, B20, B26, B27
W5_Softmax_C0	B1, B3, B9, B12, B13, B15, B16, B17, B18, B20, B22, B23, B25, B26, B28
W5_Softmax_C1	B5, B8, B14, B27, B29
W5_Softmax_C2	B2, B4, B6, B7, B10, B11, B19, B21, B24

4.2. Asset Selection using SVM Model

Tables 9 and 10 present the shortlisted assets obtained using the multi-class SVM model. Compared to the Softmax model, the SVM-based classification demonstrates a more balanced and structured allocation of assets across classes.

For the BSE 30 dataset, the SVM model consistently distributes assets across all three classes in most windows. In W1, the model assigns a moderate number of assets to each class, with a clear distinction between low, medium, and high-return assets. In subsequent windows such as W2 and W4, the model maintains this balance, although certain windows show a higher concentration in Class 2, indicating stronger identification of high-return opportunities. Notably, in W3 and W4, the majority of assets fall into Class 2, suggesting that the SVM model is more aggressive in identifying potential high-performing assets compared to Softmax.

For the DOW 30 dataset, the SVM model again shows improved class separability. In W1, assets are distributed across all three classes with a clear distinction. In W2, almost all assets fall into Class 0, indicating a cautious market phase. However, unlike Softmax, the SVM model maintains better differentiation in subsequent windows, particularly in W3 and W4, where assets are clearly split between Class 1 and Class 2. This demonstrates the robustness of the SVM model in handling varying market conditions.

The primary insight from the SVM-based asset selection is that it provides more stable and discriminative classification across rolling windows, reducing the issue of class imbalance observed in the Softmax model.

Table 9. Shortlisted Assets for BSE 30 (SVM Model)

Sheet	Shortlisted Assets
W1_SVM_C0	A10, A13, A14, A16, A27, A29
W1_SVM_C1	A1, A3, A4, A6, A7, A8, A11, A12, A15, A17, A18, A19, A20, A22, A23, A24, A28, A30
W1_SVM_C2	A2, A5, A9, A21, A25, A26
W2_SVM_C0	A1, A2, A5, A6, A7, A8, A10, A11, A12, A13, A14, A15, A17, A19, A20, A23, A24, A25, A26, A28, A30
W2_SVM_C1	A4, A9, A16, A21, A27
W2_SVM_C2	A3, A18, A22, A29
W3_SVM_C1	A1, A6, A8, A15, A21, A23, A27, A29
W3_SVM_C2	A2, A3, A4, A5, A7, A9, A10, A11, A12, A13, A14, A16, A17, A18, A19, A20, A22, A24, A25, A26, A28, A30
W4_SVM_C0	A2, A3, A4, A7, A13, A14, A19, A21
W4_SVM_C2	A1, A5, A6, A8, A9, A10, A11, A12, A15, A16, A17, A18, A20, A22, A23, A24, A25, A26, A27, A28, A29, A30
W5_SVM_C0	A2, A4, A6, A7, A8, A10, A11, A14, A16, A19, A20, A21, A22, A23, A24, A25, A27, A30
W5_SVM_C1	A3, A17, A28
W5_SVM_C2	A1, A5, A9, A12, A13, A15, A18, A26, A29

Table 10. Shortlisted Assets for DOW 30 (SVM Model)

Sheet	Shortlisted Assets
W1_SVM_C0	B2, B3, B5, B8, B11, B15, B19, B20, B22, B24, B26, B28
W1_SVM_C1	B1, B9, B14, B18, B21, B23
W1_SVM_C2	B4, B6, B7, B10, B12, B13, B16, B17, B25, B27, B29
W2_SVM_C0	B1, B2, B3, B4, B5, B6, B7, B8, B9, B10, B11, B12, B13, B14, B15, B16, B17, B18, B19, B20, B21, B22, B23, B24, B25, B26, B27, B28, B29
W3_SVM_C1	B3, B9, B15, B20, B21, B23, B24, B25, B27, B28
W3_SVM_C2	B1, B2, B4, B5, B6, B7, B8, B10, B11, B12, B13, B14, B16, B17, B18, B19, B22, B26, B29
W4_SVM_C0	B2, B10, B16, B18, B19, B20, B24, B27
W4_SVM_C2	B1, B3, B4, B5, B6, B7, B8, B9, B11, B12, B13, B14, B15, B17, B21, B22, B23, B25, B26, B28, B29
W5_SVM_C0	B4, B7, B28
W5_SVM_C1	B2, B5, B10, B13, B19, B20, B22, B23, B26, B29
W5_SVM_C2	B1, B3, B6, B8, B9, B11, B12, B14, B15, B16, B17, B18, B21, B24, B25, B27

4.3. Worst Case Omega Portfolio Optimization Using Shortlisted Assets

Tables 11 and 12 comparing the performance of Softmax and SVM-based portfolios for both the BSE 30 and the DOW 30 datasets provide a comprehensive evaluation of the proposed framework. The portfolios are constructed using assets from Class 1 and Class 2, as these represent moderate and high-return investment opportunities.

For the BSE 30 dataset, the SVM-based portfolios consistently outperform the Softmax-based portfolios across most performance metrics. The mean returns for SVM are higher in both Class 1 (0.00078) and Class 2 (0.00125) compared to Softmax. Risk measures such as standard deviation, drawdown, Value-at-Risk (VaR), and Conditional Value-at-Risk (CVaR) are lower for SVM, indicating better downside risk management. Furthermore, performance ratios, including Sharpe ratio, Sortino ratio, STARR ratio, and Calmar ratio, are significantly higher for SVM portfolios, demonstrating superior risk-adjusted performance. The Omega ratio, which captures the probability-weighted ratio of gains to losses, is also higher for SVM, confirming its effectiveness in generating favorable return distributions.

For the DOW 30 dataset, a similar trend is observed. Although Softmax achieves slightly higher maximum returns in certain cases, the SVM model delivers more consistent performance with lower volatility and reduced downside risk. The SVM-based portfolios exhibit higher Sharpe, Sortino, and Omega ratios, indicating improved stability and efficiency. Additionally, the skewness and kurtosis values suggest that SVM portfolios have more favorable return distributions with reduced tail risk in most cases.

Overall, the experimental results clearly demonstrate that the integration of SVM-based classification with worst-case Omega optimization leads to superior portfolio performance compared to the Softmax-based approach. The SVM model not only improves asset selection through better class separability but also enhances portfolio outcomes by reducing risk and improving return consistency.

The key knowledge gained from this experiment is that classification quality plays a crucial role in portfolio optimization. A more discriminative classifier, such as SVM, can significantly enhance the effectiveness of downstream optimization models, particularly in uncertain and dynamic financial markets.

Table 11. Performance Comparison: Softmax vs SVM (BSE 30)

	Class 1 BSE 30		Class 2 BSE 30	
Model	**Softmax**	**SVM**	**Softmax**	**SVM**
Mean	0.00059	**0.00078**	0.00067	**0.00125**
Median	0.00042	**0.00075**	0.00067	**0.00121**
Min	**-0.14813**	-0.06287	-0.12133	**-0.12199**
Max	**0.10055**	0.04678	**0.07990**	0.07818
Std	0.01588	**0.01185**	0.01407	**0.01377**
DD	0.01216	**0.00893**	0.01065	**0.01053**
Max Drawdown	-0.36310	**-0.22890**	-0.26566	**-0.24433**
VaR_{95}	-0.02174	**-0.01739**	-0.01970	**-0.01935**
$CVaR_{95}$	-0.03599	**-0.02853**	-0.03207	**-0.03066**
Sharpe	0.03691	**0.06576**	0.04740	**0.09045**
STARR	0.01628	**0.02731**	0.02080	**0.04061**
Sortino	0.04820	**0.08722**	0.06263	**0.11822**
Calmar	0.43816	**0.94748**	0.68871	**1.50744**
Gain/Loss	**1.04917**	1.03017	1.01736	**1.02079**
Omega	1.11434	**1.20523**	1.14317	**1.29517**
Skewness	-0.58312	**-0.39124**	-0.71636	**-0.58587**
Kurtosis	**11.77226**	3.62983	8.60188	**9.46751**

Table 12. Performance Comparison: Softmax vs SVM (DOW 30)

	Class 1 DOW 30		Class 2 DOW 30	
Model	**Softmax**	**SVM**	**Softmax**	**SVM**
Mean	0.00015	**0.00036**	**0.00063**	0.00058
Median	0.00057	**0.00082**	**0.00127**	0.00086
Min	**-0.09961**	-0.05358	**-0.13191**	-0.10991
Max	**0.07042**	0.04917	**0.14090**	0.07124
Std	0.01329	**0.00941**	0.01865	**0.01418**
DD	0.01080	**0.00678**	0.01473	**0.01201**
Max Drawdown	-0.38730	**-0.18867**	-0.41975	**-0.33045**
VaR_{95}	-0.02055	**-0.01509**	-0.02842	**-0.02069**
$CVaR_{95}$	-0.03343	**-0.02124**	-0.04521	**-0.03508**
Sharpe	0.01162	**0.03876**	0.03375	**0.04057**
STARR	0.00462	**0.01717**	0.01392	**0.01639**
Sortino	0.01430	**0.05377**	0.04272	**0.04791**
Calmar	0.10247	**0.51009**	0.40939	**0.47186**
Gain/Loss	0.93307	**0.94480**	0.91684	**0.92041**
Omega	1.03587	**1.11884**	1.10944	**1.13811**
Skewness	-0.51672	**0.15317**	**0.05119**	-0.61400
Kurtosis	**8.14300**	4.47798	10.67161	**11.07511**

4.4. Comparative Analysis

This subsection presents a comprehensive comparative evaluation of the proposed framework using graphical evidence, focusing on cumulative returns, excess returns, asset allocation patterns, and downside risk measures. The analysis highlights the effectiveness of the SVM-based classification integrated with worst-case Omega optimization.

Figure 1 illustrates the cumulative returns for the BSE 30 index under Class 1 and Class 2 portfolios. A clear distinction is observed between the two classes, where Class 2 portfolios consistently achieve superior performance compared to Class 1. The steeper growth trajectories in Class 2 indicate that the classification framework successfully identifies high-return assets. Moreover, portfolios derived using SVM exhibit smoother and more stable cumulative return paths, reflecting reduced volatility and improved consistency relative to Softmax-based portfolios. This demonstrates that improved class separability directly translates into enhanced portfolio performance.

Figure 2 compares the cumulative returns of the worst-case Omega optimized portfolios with the equally weighted (EW) benchmark for different values of the risk-aversion parameter δ. Across both BSE 30 (Class 2) and DOW 30 (Class 1), the Omega-based portfolios of the SVM model consistently outperform the EW portfolio, confirming the advantage of optimization over naive allocation. It is observed that lower values of δ (e.g., $\delta = 0.2$) emphasize return maximization, resulting in higher cumulative gains but increased variability. In contrast, higher values of δ (e.g., $\delta = 0.8$) produce smoother return profiles, indicating stronger downside protection. This highlights the flexibility of the Omega framework in controlling the trade-off between return and risk.

Figure 3 presents the excess returns of the Omega-optimized portfolios relative to the EW benchmark. The consistently positive excess return curves indicate that the proposed framework generates persistent alpha across both datasets. The magnitude of excess returns is higher for lower δ, suggesting that aggressive strategies yield greater outperformance. However, even under conservative risk settings, the portfolios maintain positive excess returns, demonstrating the robustness and stability of the optimization framework.

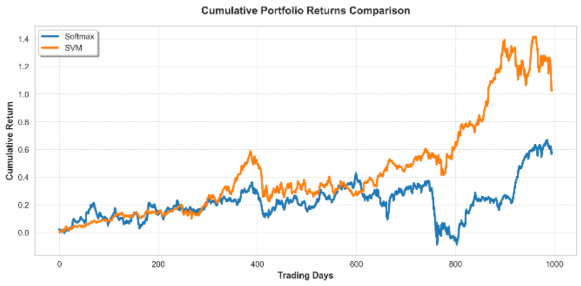
Figures 4 and 5 show the heatmaps of portfolio weights for BSE 30 and DOW 30 under Class 1 and Class 2. The heatmaps reveal that Class 2 portfolios are characterized by more concentrated and structured allocations, indicating selective investment in high-performing assets. In contrast, Class 1 portfolios exhibit more dispersed allocations, reflecting moderate-return strategies. The SVM-based portfolios display clearer and more consistent allocation patterns across time, suggesting improved stability in asset selection compared to Softmax.

Figures 6 and 7 compare the Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) at the 95% confidence level. The results indicate that SVM-based portfolios consistently achieve lower VaR and CVaR values, particularly in Class 2 portfolios. This demonstrates the ability of the proposed framework to effectively limit extreme losses. The reduction in CVaR is especially important, as it captures tail risk beyond the VaR threshold, confirming enhanced robustness against adverse market conditions.

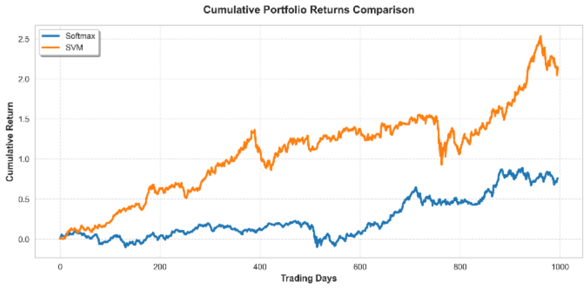
Figures 8 and 9 further analyze tail risk through CVaR-based measures. It is evident that SVM-based portfolios exhibit significantly lower tail risk compared to Softmax, with less extreme negative outcomes. The distributions are more controlled and less heavy-tailed, indicating improved resilience during market downturns. This validates the effectiveness of the worst-case Omega optimization in explicitly accounting for unfavorable scenarios.

Overall, the graphical analysis strongly supports the superiority of the proposed framework. The integration of SVM-based classification with worst-case Omega optimization not only enhances return generation but also significantly improves risk-adjusted performance and downside protection. The results demonstrate that classification accuracy plays a crucial role in portfolio construction, and when combined with robust optimization techniques, it leads to consistent outperformance across different market conditions.

The key insight derived from this analysis is that the proposed framework achieves an optimal balance between return maximization and risk control. By incorporating predictive intelligence and worst-case risk considerations, the model provides a reliable and adaptive approach for real-world portfolio management under uncertainty.



(a) Class 1



(b) Class 2

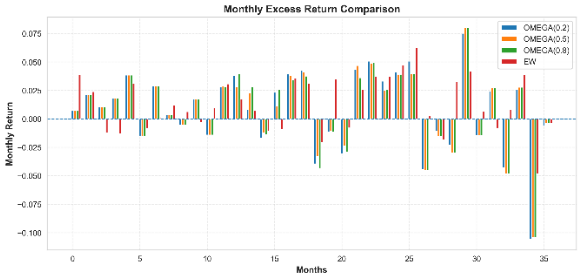
Figure 1. Cumulative Returns for BSE 30 under Different Classes

(a) BSE 30 (Class 2)

(b) DOW 30 (Class 1)

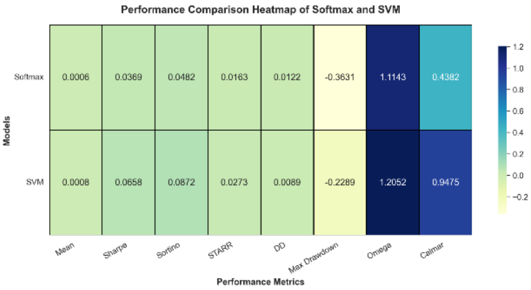
Figure 2. Cumulative returns of the Worst-Case Omega optimization model for SVM under different risk-aversion parameters ($\delta = 0.2,0.5,0.8$) compared with the equally weighted (EW) portfolio for BSE 30 and DOW 30 indices.

(a) BSE 30 (Class 2)



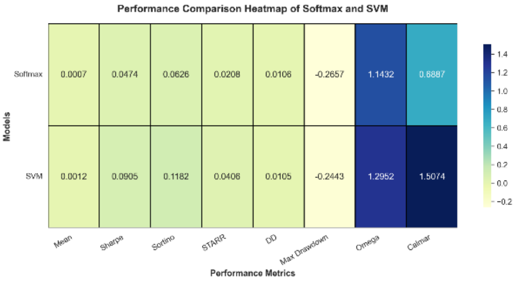
(b) DOW 30 (Class 1)

Figure 3. Excess returns of the Worst-Case Omega model for SVM and $\delta = 0.2,0.5,0.8$ compared with the equally weighted (EW) portfolio.


Performance Comparison Heatmap of Softmax and SVM

Models	Mean	Sharpe	Sortino	STARR	DD	Max Drawdown	Omega	Calmar
Softmax	0.0006	0.0369	0.0482	0.0163	0.0122	-0.3631	1.1143	0.4382
SVM	0.0008	0.0658	0.0872	0.0273	0.0089	-0.2289	1.2052	0.9475

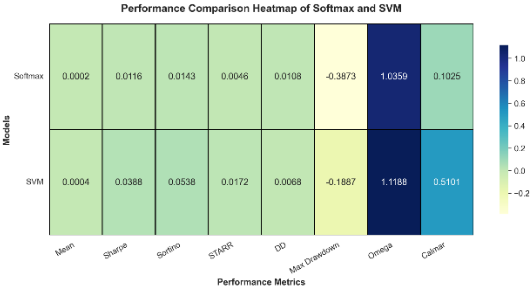
(a) Class 1


Performance Comparison Heatmap of Softmax and SVM

Models	Mean	Sharpe	Sortino	STARR	DD	Max Drawdown	Omega	Calmar
Softmax	0.0007	0.0474	0.0626	0.0208	0.0106	-0.2657	1.1432	0.6887
SVM	0.0012	0.0905	0.1182	0.0406	0.0105	-0.2443	1.2952	1.5074

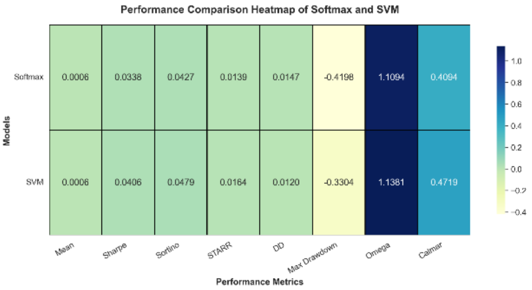
(b) Class 2

Figure 4. Heatmap for BSE 30 under Different Classes


Performance Comparison Heatmap of Softmax and SVM

Models	Mean	Sharpe	Sortino	STARR	DD	Max Drawdown	Omega	Calmar
Softmax	0.0002	0.0116	0.0143	0.0046	0.0108	-0.3873	1.0359	0.1025
SVM	0.0004	0.0388	0.0538	0.0172	0.0068	-0.1887	1.1188	0.5101

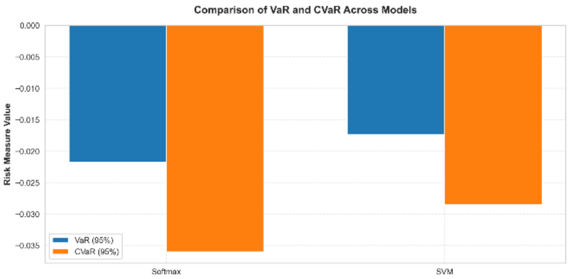
(a) Class 1


Performance Comparison Heatmap of Softmax and SVM

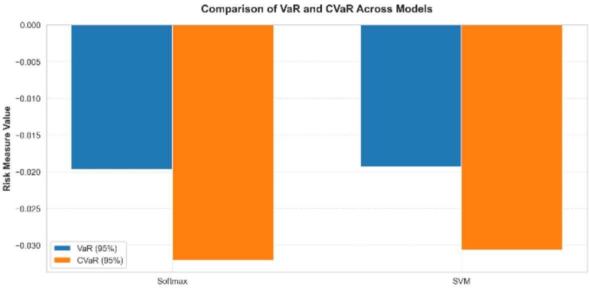
Models	Mean	Sharpe	Sortino	STARR	DD	Max Drawdown	Omega	Calmar
Softmax	0.0006	0.0338	0.0427	0.0139	0.0147	-0.4198	1.1094	0.4094
SVM	0.0006	0.0406	0.0479	0.0164	0.0120	-0.3304	1.1381	0.4719

(b) Class 2

Figure 5. Heatmap for DOW 30 under Different Classes

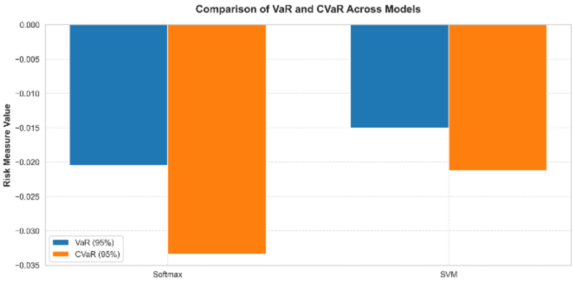


(a) Class 1

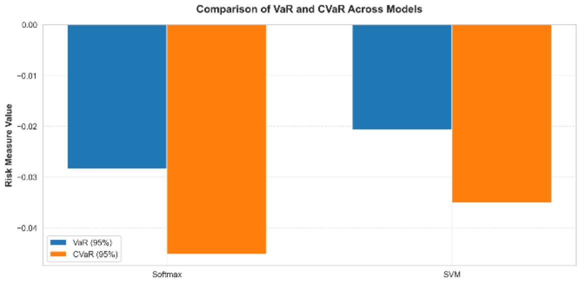


(b) Class 2

Figure 6. Comparison of Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) at the 95% confidence level for different models on BSE 30 under Class 1 and Class 2.

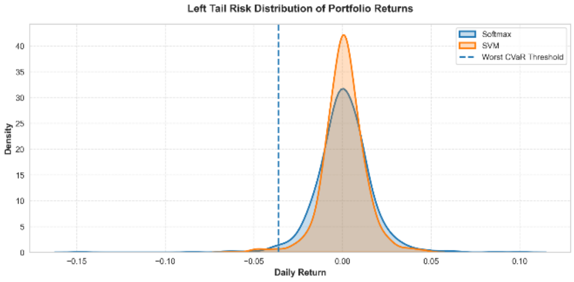


(a) Class 1

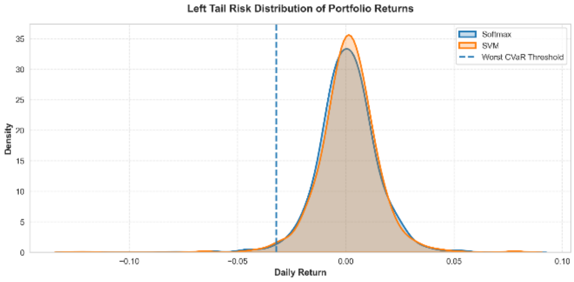


(b) Class 2

Figure 7. Comparison of Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) at the 95% confidence level for different models on Dow 30 under Class 1 and Class 2.

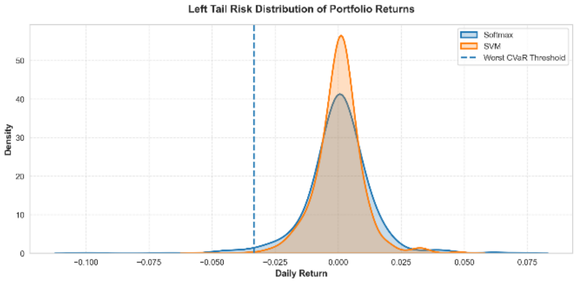


(a) Class 1

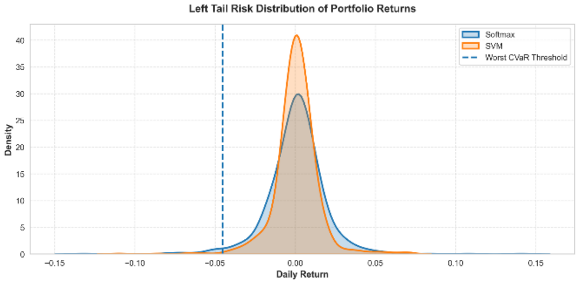


(b) Class 2

Figure 8. Tail Risk CVaR for BSE 30 under Different Classes



(a) Class 1



(b) Class 2

Figure 9. Tail Risk CVaR for DOW 30 under Different Classes

5. CONCLUSIONS

This study presents a comprehensive framework that integrates deep learning-based multi-class classification with worst-case Omega portfolio optimization for financial prediction and asset allocation. The proposed approach combines the predictive capability of neural networks with the discriminative power of Softmax and SVM classifiers, followed by a robust optimization model that explicitly accounts for downside risk and uncertainty.

The empirical findings reveal that the SVM-based classification model provides superior performance compared to the Softmax model in terms of class separability, stability, and consistency across rolling windows. By effectively identifying high-return assets, the SVM model enhances the quality of inputs to the portfolio optimization stage. As a result, portfolios constructed using SVM-selected assets demonstrate higher mean returns, lower volatility, and significantly improved risk-adjusted performance metrics.

The integration of worst-case Omega optimization further strengthens the framework by incorporating tail risk considerations into the decision-making process. The results indicate that the optimized portfolios achieve lower VaR and CVaR values, reduced drawdowns, and more stable return distributions. Additionally, the flexibility of the risk-aversion parameter allows investors to adjust the trade-off between return maximization and risk control according to their preferences.

Comparative analysis with equally weighted portfolios confirms that the proposed framework consistently generates positive excess returns, demonstrating its effectiveness in real-world investment scenarios. The findings also highlight that classification accuracy plays a critical role in portfolio construction, as improved predictive performance directly translates into better optimization outcomes.

In conclusion, the proposed hybrid framework provides a robust and adaptive approach for multi-class financial prediction and portfolio optimization. It offers significant improvements in both return generation and risk management, making it a valuable tool for investors and portfolio managers operating in dynamic and uncertain financial environments.

Future research may explore the integration of advanced deep learning architectures, such as attention-based models or transformers, as well as the extension of the framework to multi-period and multi-objective portfolio optimization settings.

Declaration of Ethical Standards

The author declares that this manuscript is original, has not been published previously, and is not under consideration for publication elsewhere. The author has significantly contributed to the research work, approved the final version of the manuscript, and agreed to its submission. All ethical guidelines related to authorship, citation, data reporting, and originality of research have been followed. All sources of information and data used in this study have been properly cited and acknowledged.

CRediT Authorship Contribution Statement

Simrandeep Kaur: Conceptualization, Methodology, Software, Data Curation, Formal Analysis, Investigation, Writing – Original Draft, Visualization, Validation, Review & Editing.

Declaration of Competing Interest

The author declares that there are no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data Availability Statement

The data used in this study were collected from publicly available financial sources, primarily Yahoo Finance and historical market index records for BSE 30 and Dow 30. The dataset covers the period from January 2015 to March 2022. The data are publicly available and can be accessed through Yahoo Finance and related market databases. The processed data supporting the findings of this study are available from the corresponding author upon reasonable request.

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